1 Introduction

In this lab you will continue to explore the RC circuit that you built during the last lab (see Fig. 1). This time, you will study the phase properties of the RC circuit response.

\[ V_{in} \]

\[ R = 1k\Omega \]

\[ C = 0.1\mu F \]

\[ V_{out} \]

Figure 1: A low-pass RC filter.

2 Measure Phase Response

Drive the input with a sinusoidal input. We can model this with a cosine function

\[ V_{IN} = V_0 \cos(\omega t), \]

as long as we decide to refer to one of the peaks as \( t = 0 \).

Connect the oscilloscope so that you can simultaneously view the input and the output of the RC circuit. You should use the scope probes from now on. This means that you have to read the vertical scope scale from the “10x” tick mark instead of the “1x” tick mark.

For a range of frequencies between 100 Hz and 100 kHz, record the phase difference between the input and the output. It will probably be easiest to calculate the phase difference based on the time difference between a certain point on the two waves. If you are sure that the scope traces have been centered on the screen, then use the rising-edge of a zero crossing for the reference point. You need to keep track of the time delay of the output relative to the input. Make a data table that looks like Table 1. You will collect data for the period of the wave, and for the \( \Delta t \) between the input and output waves as shown in Fig. 2.

As you collect this data, you will need to change the horizontal scale and probably the vertical scale for the filter output. Based on last week, what frequencies will this circuit pass? Which frequencies will it block? Be quantitative and specific.
Figure 2: $\Delta t$ is measured as shown.

Table 1: Sample data table

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\Delta t$</th>
<th>$f$</th>
<th>$\omega$</th>
<th>$\delta \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01 s</td>
<td>-</td>
<td>100 Hz</td>
<td>628 rad/s</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
3 Calculating $\delta \phi$

You can think of the phase of a sine or cosine function as how far it has gone through one cycle. Because these functions are cyclical (or periodic) phase can really only have a value between 0 and $2\pi$. Phase is measured in radians so we will have to determine how to measure a time difference between two waves and convert it into a phase difference. Note, it only really makes sense to talk about the phase difference between two waves of the same frequency. This is the case here, so we can safely continue.

Remember that the angular frequency $\omega$ of a wave is measured in radians per second. Therefore, if we know $\omega$ for the two waves then we can use the fact that

$$\delta \phi = \omega \Delta t,$$

where $\Delta t$ is the time shift between the two waves. This calculation will give $\delta \phi$ in radians. To convert to degrees, we would follow

$$\delta \phi_{\text{degrees}} = \frac{180}{\pi} \omega \Delta t \quad (1)$$

$$= \frac{180}{\pi} 2\pi f \Delta t \quad (2)$$

$$= 360f \Delta t, \quad (3)$$

where $f$ is the wave frequency in Hz.

4 Results

Report your results in a plot of $\delta \phi$ vs. $f$. Use degrees for your units of phase difference, and report $f$ on a logarithmic scale (hint: use Excel to create a column of $\log(f)$ and plot that column).

Discuss what you observe in terms of lead and lag. Describe the output relative to the input, does it lead or lag? How does the amount of lead or lag depend on frequency?
Propagation of Error

The rules for propagating an uncertainty depend on whether the quantity that is being computed is the result of a sum, difference, product, or quotient operations. The following rules guide your calculations.

If \( q = x + \cdots + z - (u + \cdots + w) \), then the uncertainty in \( q \) is given by

\[
\delta q = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2}.
\]

If \( q = \frac{x \cdots z}{u \cdots w} \), then

\[
\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2}.
\]