Quantum Mechanics Spin Lab

Name: __________________________

Open the PhET simulation titled “Stern-Gerlach Experiment,” the easiest way to find this is to google “phet stern gerlach.” From the PhET website, click the “Run Now!” button. Play around with the following controls and note what they affect.

- angle
- up/down choices
- autofire
- spin orientation checkbox

1. What does the pie chart at the bottom of the screen indicate?

2. Send an unpolarized beam of particles (random xz) through a Stern-Gerlach magnet oriented in any direction. How does the beam split at the magnet? Does the splitting ratio depend on the orientation? Explain why this is the case.

3. Why does the splitting ratio fluctuate?

4. What one-magnet configurations give 100% spin-up output beams?

5. This requires two magnets: send an unpolarized beam of particles through a z magnet, then send the +z beam through the x magnet. How does the beam split at the x magnet?

6. This requires three magnets: first send an unpolarized beam of particles through a z magnet, then send the +z beam through an x magnet, then send the +x beam or the -x beam through another z magnet. Is there anything strange about the output? What was the original state of the beam and is that state preserved throughout the system?

In class we will show that the spin operators are represented by the following matrices:

\[ S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]  
\[ S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]  
\[ S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]  

We will also show that spin states can be represented in the z-basis which has two basis vectors:

\[ |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]  
\[ |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]
7. \(|\uparrow\rangle\) and \(|\uparrow\rangle\) are the eigenstates of the \(S_z\) operator, what are their \(S_z\) eigenvalues?

8. Next you will check the normalization of several similar states. Note, to check the normalization, you need to know that the dual vector \((\rightarrow|\) is given by converting the column vector to a row vector and taking the complex conjugate. For Example:

\[
(\rightarrow| = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

and

\[
(\uparrow| = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}
\]

then in order to check the normalization, evaluate the inner product between a vector and its dual:

\[
(\uparrow|\uparrow) = 1
\]

Show that the following are the normalized eigenstates of the \(S_x\) and \(S_y\) operators (remember, this requires checking that they are normalized and that they are eigenstates). Also, find the eigenvalue of each. If the eigenvalue of \(S_x\) is positive, we say that state has +x spin.

Eigenvalues of \(S_x\):

\[
|\rightarrow\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[
|\leftarrow\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}
\]

Eigenvalues of \(S_y\):

\[
|\uparrow\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}
\]

\[
|\downarrow\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}
\]

9. Using this formulation, check the probabilities that you observed in the simulation. For example, to calculate the probability of measuring a particle to have +x spin if you know the incoming particle is in the state \(|\uparrow\rangle\), evaluate \(P = |(\rightarrow|\uparrow\rangle)|^2\). Check the other probability that you simulated as well (the probability of measuring +z spin when you know the state is \(|\rightarrow\rangle\)).