Profile Measurements of Focusing Laser Beams

1. Abstract

Focused laser beams are commonly used in characterizing the intensity dependence of the optical properties of various materials. In order to compare the experimentally measured intensity dependence with theoretical predictions, it is vital that the focused beam maintains a characteristic Gaussian profile. However, our laser beam’s profile is frequently distorted by the optics used in these experiments. This motivates directly measuring the beam’s profile both before and after it is focused. Measuring a focused beam is tricky, and to this end we employ the knife-edge method to see how closely our focused beam compares to the expected Gaussian profile. Verification that our implementation of the knife-edge method works is given by a strong linear relationship between our observed data and a standard Gaussian integral, for the most part strong nonlinear direct fits of experimental response curves, as well as nearly reproducing the 1/e radius measured by a professional CCD camera.

2. Background

In order to understand why our experiments require focal spot measurements, one must first understand how the spatial profile of a beam of light interacts with our laboratory environment. Several different topics fall under this umbrella. First, we must understand what kind of object a Gaussian beam is. Then we develop an understanding of exactly what focusing a beam through a lens does. We will finish by examining refractive index, electromagnetic...
modes in a cylindrical cavity, and capillary waveguides, in order to see why our equipment is sensitive to focused beam behavior.

**Gaussian Beams**

A beam is any copropagating group of photons. To be useful, beams are collimated, forced to not spread out significantly more than inherent diffraction. It is standard to define the direction of beam propagation as the z-axis and then to conveniently define the x and y axes for the specific application. Our experiments are all carried out on a level optical table with the laser beam traveling parallel to the table surface. For our purposes, the x-axis is defined to be parallel to the table and perpendicularly to the left of beam propagation, and the y-axis points up and away from the table, as pictured in Figure 1.

![Figure 1: Illustration of which way our coordinate axes point](image)

A Gaussian beam is one in which the transverse intensity distribution within the beam follows a Gaussian function. That is, the area density of the photons is the same bell curve when projected on either the x- or y-axis. Provided that there are no concentrations of different wavelengths within the beam, this means that the energy, power, and flux profiles are also described by Gaussian functions. The flux of such a beam can be mathematically represented as
\[ \Phi(x, y) = \Phi_0 e^{-\beta^2((x-x_0)^2 + (y-y_0)^2)} \]  

Eqn. 1

where \( \Phi_0 \) is the photon flux at the peak, \( x_0 \) and \( y_0 \) give the coordinates of the peak and \( 1/\beta \) is the radius at which the flux has decayed to 1/e of the original peak value. This idea of a 1/e radius deserves elaboration. Since the Gaussian function extends out to infinity in the transverse directions, there is no absolute boundary to be measured, and any radius is necessarily defined as the radius at which the flux falls to a specified fraction of its peak value. Of course experimentally, there are absolute cutoffs; if nothing else the opening of the laser’s optical cavity blocks the beam. However, provided that the truncation happens far away from the center where the beam is dim, the deviation from a mathematical Gaussian will be minimal, and a fractionally defined radius will be more useful for talking about the energetic area of the beam.

**Behavior of Focusing Beams**

In a simple treatment of beams, one uses rays to describe the propagation of the wave and changes the relative angles of these rays when the light passes through a lens. This is the basic approach in geometric optics, and is a good approximation if diffraction can be ignored and one is not concerned with beam’s shape near the focal point. In our experiments, however, behavior near the focal points of our lenses is very important, so we have to treat focusing beams more carefully. The motivation for modifying the geometric interpretation is quite simple. If the photons were really converging to exactly the focal point, even the dimmest pen laser could create a point of infinite energy density, forming plasma. Clearly this does not happen, so something beyond the geometric model must limit the extent to which the beam focuses. What is observed is that each differential spatial piece of the beam is inherently bent towards a slightly different position in the focal plane, and the original profile becomes its spatial Fourier transform at the focal plane. A wave of infinite transverse extent, i.e. an infinite plane wave, is the only
type of function that will Fourier transform to a delta function (that worrisome infinite point). Such waves are impossible to generate experimentally, which averts the infinite energy density catastrophe. More generally, as the spatial features of a wave become smaller (specifically higher in spatial frequency), their Fourier transforms become broader (lower in spatial frequency). Qualitatively, smaller objects are larger in the focal plane. There are a few mathematical functions, which are scaled but not distorted in overall shape by Fourier transformation, and one of these is the Gaussian. This is a major reason for trying to generate and maintain a Gaussian profile. The Fourier transform, or for that matter any level of focusing, of a Gaussian is simply another Gaussian\(^1\), which means that the beam profile will not change significantly, regardless of how many lenses it is sent through.

To describe the size of a focusing beam, we use two main quantities, beam waist and Rayleigh range. Beam waist is the same as beam radius but specifically refers to the \(1/e\) radius. The Rayleigh range is the distance over which the beam waist is less than or equal to \(\sqrt{2}\) times the minimum waist. Rayleigh range is inherently a diffraction-dependent quantity and can thus be applied to either a focal spot or a freshly collimated beam. For lasers, there is a Rayleigh range associated with the beam waist exiting the optical cavity. The equation for the Rayleigh range of a Gaussian beam is

\[
z_R = 2 \frac{\pi \omega_0^2}{\lambda}
\]

Eqn. 2

where \(\omega_0\) is the initial beam radius (at the focal spot or point of collimation) and \(\lambda\) is the wavelength of the light in the beam. The waist of a Gaussian beam at the focal plane of a lens is given by
\[
\omega_0' = \frac{1}{2} \frac{\lambda f}{\pi \omega_0} \left( \frac{1}{1 + \frac{f^2}{z_R^2}} \right) = \frac{1}{2} \frac{\lambda f}{\pi \omega_0} \left( \frac{1}{1 + \frac{f^2 \lambda^2}{\pi^2 \omega_0^4}} \right)
\]

Eqn. 3

where \( \omega_0 \) is the beam radius incident on the lens, \( f \) is the focal length of the lens, \( \lambda \) is the wavelength, and \( z_R \) is the Rayleigh range of the incident beam. These equations are adapted from a laser physics book by Milonni & Eberly,\(^2\) who derive waist and Rayleigh range in terms of \( \frac{1}{e^2} \) rather than \( 1/e \). Their definitions are more conventional but did not work with the experimental methods that we were already using. The resulting differences are the factor of 2 in the Rayleigh range and the factor of 1/2 in the focal waist equation. Waist and Rayleigh range at focus are important values whenever the focal spot must fit into small holes, as is the case for spatial filters and capillary wave guides, which are discussed later.

**Refractive Index**

Refractive index is the property that determines how fast light travels though a material. A material’s index, \( n \), is commonly defined as the ratio of the vacuum speed of light to the measured propagation speed. In terms of more theoretically fundamental properties, index is a function of the material’s dielectric constant (\( \kappa \)) and relative magnetic permeability (\( \kappa_m \)).

\[
n = \frac{c}{v} = \sqrt{\kappa \cdot \kappa_m}
\]

Eqn. 4

Light waves traveling through a material with a high refractive index are traveling significantly slower than they would in a vacuum. This can be thought of as the high index medium “holding on to photons more tightly.” At an interface between high and low index media, the light has to have enough momentum perpendicular to the interface to escape the high index material. Thus
high index materials have the ability to trap light under certain conditions. At an interface between media of different refractive indices, incident light’s propagation direction changes. Relative to the surface normal of the interface, this transformation is usually well described by Snell’s Law (eqn. 5).

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]  

\textbf{Eqn. 5}

**Behavior of Light in a Cylindrical Cavity**

Much like a flute resonating at low C, cylindrical cavities have preferred propagation modes for light traveling through them. The simplest and easiest of these modes to generate are Gaussian. The fact that most lasing crystals are themselves cylindrical cavities outputting a roughly Gaussian profile is the most fundamental reason for basing models on that energy distribution; that Gaussian profiles do not deform from focusing is simply a big added bonus. These modes are set by the boundary conditions imposed by the walls of the tube. Because these boundary conditions apply independently to the transverse electric and magnetic fields of the light, these modes are described by two different indices, corresponding, respectively, to the number of nodes in the electric (radial nodes) and magnetic (circular nodes) fields’ standing wave patterns. Figure 2 illustrates several of these modes.
The important thing to realize is that the propagation modes have a wide array of shapes and that, when combined in the correct proportions, many different patterns can be created from them. In other words, you can fit a wide variety of beam profiles into a cylindrical tube so long as their constituent modes also fit into the tube.

**Capillary Wave Guides**

Capillary wave guide is a catchall term that refers to fiber optic cables and similar devices. Waveguides work by creating a cavity bounded by a change in refractive index. The two basic regions of a waveguide are the core and cladding. The core is the relatively high-index central portion through which light is guided, and the cladding is the relatively low-index portion that surrounds the core. The basic explanation is that, when light enters the core at a low enough angle to exceed the critical angle of the core-cladding interface, it is totally internally reflected and thus “trapped” in the core. To properly model wave guiding, one has to take into account the actual electric and magnetic fields of the propagating light and their restriction by the core-
cladding boundary, but that level of detail is unnecessary for our project. We do however need to acknowledge two basic results of the E&M interpretation.

Both of these technicalities lead to what is broadly referred to as “coupling efficiency,” which is a measure of how much light energy is lost due to entering and propagating through a wave guide. First, because the core-cladding boundary is simply an index mismatch between otherwise translucent materials, a measurable fraction of the light always escapes into the cladding, contrary to our total internal reflection model. This lowers coupling efficiency because now a fraction of the light escapes along any given length of waveguide. This specific effect is often referred to as “lossiness,” and is an unavoidable property of the wave guide itself. Second, most waveguides are actually cylindrical cavities with semipermeable boundaries and therefore strongly prefer to propagate the cylindrical Gaussian modes pictured in Figure 4. Additionally, wave guides are more effective for lower order modes. The consequence to coupling efficiency is that an incident beam will not be able to fully stimulate its constituent Gaussian modes and undergoes measurable attenuation as it is coupled into the core.

Since we want our beam to always be a Gaussian spot (lowest order mode), we have to be extra careful to excite only that mode of the waveguide. This condition is even stronger than simply minimizing coupling efficiency. Most fibers can propagate profiles that are significantly non-Gaussian and thus deviate from the Gaussian-profile assumption central to predictions for many of our other experiments. Therefore, the only way to ensure a Gaussian profile in our wave guides is to make sure that it is still strongly Gaussian when it is focused into the wave guide’s core.

3. Methods

Equipment
A large number of interesting instruments are necessary to properly prepare for and conduct my experiments. Several of them are programmable and have been automated to improve data quality. The following paragraphs refer almost exclusively to equipment enumerated in Figure 3 (on page 12). This is the layout of the parts of the optics table which effect my work.

Our source of photons is a 10Hz pulsed Neodymium-doped Yttrium-Aluminum-Garnet (Nd:YAG) pump laser. The lasing crystal produces 1064nm infrared photons, and two subsequent harmonic generating crystals convert the vast majority of those photons first to 532nm green and then to 355nm ultraviolet. The beam exiting the pump laser is thus primarily ultraviolet with some residual infrared and green, with a visible radius of about 3mm. This beam is immediately sent into an optical parametric oscillator (OPO), which utilizes a BBO (Barium Borate) crystal. This crystal converts the original 355nm photons to a significantly different wavelength, called the “signal,” which depends on the angle of the crystal lattice relative to the beam axis (a.k.a. the direction of the photon momentum). In addition to the signal, a longer-wavelength beam, called the “idler,” is also created. The BBO crystal’s angle can be tuned to output idler wavelengths in excess of 2000nm, and this is how we can generate any wavelength necessary for our experiments.

The incident fluence necessary to drive the OPO crystal is extremely high, which results in a dangerously bright output beam. In order to save our eyes and equipment, the first major device after the OPO is a high power attenuator. Anyone who has worn sunglasses is familiar with dimming bright light, but this kind of attenuator works very differently. At the power levels exiting the pump laser, filters, which absorb some fraction of incident light, would be incinerated. A safer way of removing large numbers of photons is to pass the beam though a
series of glass plates, which are rotated to highly glancing angles with respect to the z-axis. Glass is essentially translucent and has a higher index of refraction than air, but at very high angles relative to the plate’s surface normal, the percent of light reflected suddenly becomes very high.* The whole apparatus is then simply encased in a big beam-blocking box, so that any reflected light is absorbed. This is how the attenuator works without forcing anything that the beam is transmitted through to absorb too much power.

Another major obstacle is that the beam coming out of the OPO is significantly heterogeneous in wavelength. Several factors contribute to this. First, the BBO crystal is not 100% efficient, so there are some residual wavelengths from the pump beam. More importantly, the OPO outputs the longer idler wavelength in addition to the signal, and we often need the idler. The end result is that, when producing far infrared output wavelengths, the beam is contaminated by 4 wavelengths other than the desired one: 355nm, 532nm, 1064nm, and the signal. Though the pump wavelengths can often be directly filtered without affecting the desirable parts of the beam, the signal usually cannot be. To eliminate the signal, we use a Pellin Broca prism. The defining characteristics of the Pellin Broca prism are high dispersion and long optical path length. Dispersion is the tendency for a material’s index of refraction to change with wavelength. Thus originally copropagating beams of different wavelengths exit a high dispersion prism with a significant relative angle between their propagation directions. Then all one has to do is align the desired beam to exit straight down the table, and, given sufficient propagation length, the unwanted beam completely separates, and can be completely blocked.

Now that we have a single-wavelength beam, we need to make its profile as Gaussian as possible. The lasing cavity preferentially generates a first-order Gaussian mode and has a built

* This, by the way, is a vivid illustration that Snell’s Law is not universal.
in Gaussian filter, so the output beam is at least close to spatially Gaussian. However, there are still usually small imperfections in the form of jagged or fuzzy features in the energy profile. These types of imperfections are removed by a method called spatial filtering. As mentioned earlier, beams take the shape of their spatial Fourier transform at the focal plane of a lens. In a Fourier transform, small spatial features like our jagged areas tend to fall further away from the z-axis, while large ones like our Gaussian spot tend to fall closer to it. In fact, the Gaussian portion of our beam is still concentrated as a Gaussian spot of predictable radius centered on the z-axis. A spatial filter is simply a very small circular aperture whose radius is somewhat larger than that of the focused Gaussian. Major optical equipment manufacturer, Newport, suggests an optimal aperture diameter of

\[
D_{opt} = \frac{f \lambda}{\omega_0}
\]

Eqn. 6

where \( f \) is the focal length of the lens, \( \lambda \) is the beam wavelength and \( \omega_0 \) is the \( \frac{1}{e^2} \) beam waist in the focal plane of the lens (twice the 1/e waist). Once centered on the z-axis, the aperture is small enough that most of the non-Gaussian spatial features are blocked. When the beam is recollimated on the other side of the aperture, the profile is usually clean enough to be treated as Gaussian. However, this piece of equipment causes its own set of problems because, being a pinhole, the light coming out of it has a bull’s eye diffraction pattern in it.

* This number is a guaranteed over-estimate of the main beam’s \( \frac{1}{e^2} \) diameter. Under normal experimental conditions the \( \frac{1}{e^2} \) waist equation can be simplified by noting that \( \frac{f^2}{z_R^2} \) is small.

Using Eqn. 3 in terms of \( \frac{1}{e^2} \) diameter, \( D_0' = 2 \frac{\lambda f}{\pi \omega_0} \left( \frac{1}{\sqrt{1 + \frac{f^2}{z_R^2}}} \right) < \frac{2 \lambda f}{\pi \omega_0} < \frac{\lambda f}{\omega_0} \)
The system we use to measure pulse energies is an RM660 Universal Radiometer with two RJP-465 silicon energy detectors. The RJP-465’s actually detect the light while the RM-6600 is a command module that orchestrates the detectors and synchronizes data acquisition. The detectors can be set to one of 6 energy sensitivity ranges by sending commands RA1 through RA6 to the RM-6600. There are several different data collection schemes in which the RM-6600 can work, varying in quality and speed of data collection. For our purposes, we used the ASCII dump mode, which sends energy readings to the data buffer in ASCII format (i.e. the digits are formatted as plain text). One might wonder why one would ever want characters instead of actual numbers. The reason is that it allows information about the detector’s status to be easily synchronized with data collection. Specifically, a detector can send the reading “OR” which means that the pulse that hit it exceeded its sensitivity range. ASCII dump energy readings are in a format similar to scientific notation, where, depending on the RJP-465’s sensitivity range, a reading can be represented as either “###.#E##”, “##.##E##”, or “#.###E##”, where the # sign represents a digit or a space when it would be an extraneous leading zero. A detailed description of my computer code that automates the energy meters and other equipment is available in Appendix II.
Beam Profiling

In the background section of this paper, there was much to do about how beams interact with lenses and wave guides and how these interactions drive us to need a Gaussian beam. Both the laser and spatial filter are designed to achieve this goal. Unfortunately, the Gaussian-looking beam that comes out of the spatial filter often isn’t very Gaussian, especially if the laser is working poorly or if something in the beam path is aligned improperly. The CCD camera is helpful for verifying the quality of the beam, but, as I alluded to earlier, what look like small imperfections in the original beam can be very pronounced in the focal plane. Additionally for our z-scan project, we need to be able to precisely locate and measure the focal spot. Our CCD camera is not designed with the sub-micron resolution necessary to measure focal spots, so we needed a totally different way of imaging and quantitatively analyzing the beam near the focal plane. We used the knife-edge method to solve this problem.\(^5\)
The knife-edge procedure is fairly simple. The necessary materials are a focusing lens, a collecting lens, two submicron-resolution motion stages, a razorblade, and an appropriate light detector: energy for a pulse signal, and illumination/power for a continuous wave signal. The method will be described in terms of energy, but the same analysis applies for a continuous wave signal, provided that every instance of the word “energy” is replaced with “power.” First, the motion stages are attached to each other and bolted to the table, such that one moves in the z-direction and one moves in the y-direction. The razor blade is attached to the stages such that the sharp edge is parallel to the table and pointing up. This micron-range translating razorblade setup is inserted at the approximate z-position where the focal plane is to be formed. Next, the lens is placed such that the focal point is about at the z-position of the razorblade, and the collector and detector are aligned such that the detector can read the whole beam. The beam should be attenuated to the maximum reasonable brightness for the detector that is to be used. Before taking any data, it is necessary to scan through the beam manually and find appropriate starting and ending points for the automated blocking scan. This range should then be divided up into a couple hundred increments in order to get a smooth, detailed knife-edge scan. My data contains about 300 points, but it may be possible to get acceptable data with fewer. What must be checked is whether the increment gets acceptable resolution at the peak of the beam. As a guideline if the energy at half blocking (razorblade edge right on the peak) is 50 pJ, the readings one increment to either side should be different by less than 5 pJ.

With boundaries and increments sorted out, data can now be taken. First, a reading is taken of the beam completely uncovered. This gives the total energy of the beam. Next the

* Depending on the specifics of the setup, the collecting lens may not be necessary. In my setup, I had to use such a strong focusing lens that the beam was too big by the time it got to the detector, so I had to insert a small lens to collect the light.
razorblade is incrementally moved upwards. An energy reading and corresponding blocking position are recorded for each incremental move, until the beam has been satisfactorily blocked.

Our original analysis of this data followed a procedure set out by Khosrofian and Garetz. Each data point represents the spatial integral of all unblocked parts of the beam. So the points should be modeled by the Gaussian integral

\[
E(y_b) = \int_{-\infty}^{y_b} \int_{-\infty}^{\infty} I_0 e^{-\beta^2[(x-x_0)^2+(y-y_0)^2]} \, dx \, dy = I_0 \frac{\sqrt{\pi}}{\beta} \int_{-\infty}^{y_b} e^{-\beta^2[(x-x_0)^2+(y-y_0)^2]} \, dy
\]

Eqn. 7

where \( y_b \) is the location of the blocking razorblade, \( E(y_b) \) is the energy read by the detector, \( I_0 \) is the peak intensity of the Gaussian beam profile, and all the other variables are the standard Gaussian parameters from equation 7. For analysis purposes, this is divided by the total energy of the beam. Theoretically, the unblocked energy is

\[
E_{total} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_0 e^{-\beta^2[(x-x_0)^2+(y-y_0)^2]} \, dx \, dy = I_0 \frac{\beta^2}{\pi}
\]

Eqn. 8

corresponding to \( y_b \to \infty \). Dividing by this full, unblocked energy reading conveniently gets rid of the as yet unknown \( I_0 \) from equation 11, leaving

\[
\tilde{E}(y_b) = \frac{\beta}{\sqrt{\pi}} \int_{-\infty}^{y_b} e^{-\beta^2[y-y_0]^2} \, dy
\]

Eqn. 9

The resulting normalized energy curve is different from a standard Gaussian integral

\[
N(y_s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y_s} e^{-\frac{x^2}{2}} \, dx
\]

Eqn. 10

only by the \( \beta \) factor in the exponent. Thus a linear relationship is derived from setting these exponents equal to each other.
\[ y_b = \frac{y_s}{\beta \sqrt{2}} + y_0 \]  

Eqn. 11

Values for \( N(y_s) \) are tabulated or readily calculated. Additionally, Khosrofian and Garetz calculated a closed-form, algebraic approximation for this standard curve

\[ N(y_s) = f(y_s) = \frac{1}{1 + e^{(a_0 + a_1y_s + a_2y_s^2 + a_3y_s^3)}} \]  

Eqn. 12

where \( a_0 = -6.71387 \cdot 10^{-3} \), \( a_1 = -1.55115 \), \( a_2 = -5.13306 \cdot 10^{-2} \), and \( a_3 = -5.49164 \cdot 10^{-2} \).

The utility of all this is that since both curves are unity-normalized integrals of a Gaussian curve, they should in theory have equal values for pairs of \( y_b \) and \( y_s \) from equation 11. Thus we can use Maple to numerically solve \( \tilde{E}(y_b) = f(y_s) \) for \( y_s \) on all of our data points, giving a set of correlated \( y_b \) and \( y_s \) from the experimental data. For measurements of a perfect experimental Gaussian, these \( y_b \) and \( y_s \) values would follow equation 11 exactly. Thus running a linear regression on the set of correlated \((y_b, y_s)\) points will give values for \( \beta \) and \( y_0 \), and a least squares analysis will give a measure of how Gaussian the data actually is. If this \( R^2 \) value is significantly less than one, the beam is not making a sufficiently Gaussian profile at focus, and the beam must be realigned. Exactly what constitutes a significant deviation from one was not well addressed in the original paper. However, so long as we get a consistently straight line for \((y_b, y_s)\) points and the radius is close to the value predicted by equation 3, we can be confident that the beam is forming the Fourier transformed spot at focus, which is all we really need to know.

Due to hardware conflicts, I was never able to get the equipment to take an energy profile for the pulse laser beam. The knife-edge method has however been tested with slightly different equipment than will be used in the actual experiment, and this has been sufficient to verify that the method gets reasonable results. The Nd:YAG laser has had many historical malfunctions, so,
in order to have a consistent signal for profiling, I retrofitted the department’s green laser pen pointer (see fig. A1). This consisted of making a dummy cell, wiring it to a bench top DC power supply, strapping this all together with electrical tape, sticking it on an optics table mount, and being careful never to supply more than 3V (aka equivalent to the intended 2 AAA batteries) and 25mA (it stops drawing current at 20mA). This being a continuous wave signal, I had to use one of the department’s Vernier LS-BTA Light Sensors.

For every response run the same starting and ending positions were given to the linear motion stage, and it was directed to move at 0.002 mm/s. The illumination meters cannot be directly synchronized with the stages, so the raw data that to work with was illumination versus time taken by the illumination meter at 20 Hz. This lead to the mild piece of guess work of identifying in the illumination meter data the sequences of exactly 300 data points over which the power reading was changing and then lining this series up with the known starting and ending points. Given the stage velocity, total time to cover those 300 points, starting and ending positions and a resulting increment of 0.0001 mm, a position was assigned to each illumination measurement. For each run, the beginning of the 300-point series was uncertain by 1-3 data points, so this guess work in the data analysis contributed an additional uncertainty of +/-0.0003 mm to \( y_0 \), 1% of the length that was scanned in detail. The resulting percent error in \( \beta \) should be even less, given that the steepness of the curve is self-consistent except for a couple of manually taken points at the periphery of the continuous scan. Those peripheral points were taken at much greater increments in order to get a sense for how the curve was trailing off. This is how my reported response curves were generated.

Normalization was done relative to the unblocked beam illumination. The actual correlation to the standard curve is preformed by Maple. The array of normalized response
points is compared via the “map” command to the standard curve function, and a list of correlated standard curve positions is thereby generated. That set of standard curve points is then imported into Excel, where a linear curve fit is run for the pairs of \( y_b \) and \( y_s \), which gives the \( y_0 \) and \( \beta \) parameters as per eqn. 11. Other than the equipment used for a light source and detectors, the current level of implementation is identical to that for the Nd:YAG laser.

I also did very recently used a different paper’s analysis technique\(^6\) to extract beam radii and center positions with uncertainties. The paper really only has one main point – supported by extensive error analysis – performing a direct nonlinear fit of the response data to an error function is more accurate than numerically differentiating and fitting to a Gaussian (which is not the method I have already fully implemented). More importantly for my project, this method generates direct uncertainties by fitting to \( y_0 \) and \( 1/\beta \). I did have to do some additional work beyond the paper to perform this fit.

I used LoggerPro’s fitting routine. This was a problem because my curves are cumulative distribution functions and thus have no closed form. The error function approximation used by Veshapidze, et. al. is difficult to implement, so I had to generate my own. My normalized data sets can be written in terms of statistical functions.\(^7\)

\[
\tilde{E}(y_b) = \frac{\beta}{\sqrt{\pi}} \int_{-\infty}^{y_b} e^{-\beta^2 (y-y_0)^2} dy = D(\beta(y_b - y_0))
\]

\[
= D\left( \frac{y_b - y_0}{\omega_0^\prime} \right)
\text{ Eqn. 13 }
\]

\[
= \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{y_b - y_0}{\omega_0^\prime} \right) \right]
\text{ Eqn. 14 }
\]

Good algebraic approximations to the cumulative distribution function, \( D(x) \), are hard to come by, so I just inserted the Taylor series of the error function expanded about \( y_0^8 \) into Eqn. 14. In
order to get a curve that approximates $D(x)$ for all but the top and bottom 2-2.5% of its range, I had to use 7 terms in the Taylor series. Therefore my algebraic approximation was

$$\tilde{E}(y_b) = \frac{1}{\sqrt{2\pi}} \left[ 1 - \text{erf} \left( \frac{y_b - y_0}{\omega'} \right) \right]$$

$$\approx \frac{1}{2} \left[ 1 - \frac{2}{\sqrt{\pi}} \left( \frac{y_b - y_0}{\omega'} \right) - \frac{1}{3} \left( \frac{y_b - y_0}{\omega'} \right)^3 + \frac{1}{10} \left( \frac{y_b - y_0}{\omega'} \right)^5 - \frac{1}{42} \left( \frac{y_b - y_0}{\omega'} \right)^7 \right]$$

Fitting the normalized data to Eqn. 15 by parameters $y_0$ and $\omega'$ with LoggerPro. I got a similar set of parameters to those from the Khosrofian and Garetz analysis, but with error bars.

4. Results and Discussion

Four response curves were measured for the pen laser (Figure 4).
Figure 4: These are the graphs of normalized intensity of the pen laser with respect to blocking position of the razor blade. They are all very smooth, suggesting that background noise was well controlled in this experiment. “Near-Focus Points” 1,2,&3 correspond respectively to z=5.44, z=6.44, & z=7.44.

The first one was for the unfocused beam. This run was necessary to gauge whether my implementation of the knife-edge method was even working. Khosrofian & Garetz’ knife-edge
analysis calculated an unfocused beam radius of 0.334mm. The knife-edge analysis of Veshapidze, et. al. calculated an unfocused beam radius of 0.358 +/- 0.015 mm. Our CCD camera,* which is used in a different experiment, can take full beam maps for macroscopic beams and from that calculate a knife-edge radius, sadly without an error estimate. That camera radius for the unfocused beam in the y-direction was 0.322 mm. Without an uncertainty for the camera measurement I cannot say that my implementation reproduced the professional camera measurement., I can say that my deviation was less than 6.5%. The other three data sets are for the beam near focus. When I calculated the theoretical Rayleigh range at focus, it was on the order of millimeters, so I was able to start the blade very close to the focal spot simply by eyeballing it with a business card. The runs are each 1mm apart along the z-axis, progressing in -\hat{z}. The regressions and a summary of the direct cumulative distribution function fits are given in Figure 5 and Table 1 on the following pages. Although it is not entirely apparent in Figure 4, there were some sharp jags in the z=5.44mm data, which suggests that the laser pen’s output power may have been unstable. Initial attempts to correct this by adjusting the amplitude of the fitting equation were unsuccessful, and so I must simply report the astronomical error that LoggerPro calculated, even though it appears to be a significant over estimate of the error.

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* Coherent’s BeamView Analyzer 2.4.4 software running a LaserCam II ½”
Standard to Profile Regression
Unfocused Beam

\[ y = 0.2358x + 4.2967 \]
\[ R^2 = 0.9986 \]

Radius = 0.334 mm

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Standard to Profile Regression for
Near-Focus Point #1

\[ y = 0.012400x + 4.328 \]
\[ R^2 = 0.9990 \]

Radius = 0.0175 mm
Figure 5: Regression curves for the four different measurement points. $R^2$ values were consistently low, and outliers were at the ends, where we expect the addition of background noise to start having a significant effect. “Near-Focus Points” 1, 2, & 3 correspond respectively to $z=5.44$, $z=6.44$, & $z=7.44$. 

**Standard to Profile Regression for Near-Focus Point #2**

$y = 0.009837x + 4.328$

$R^2 = 0.9962$

**Radius = 0.0139mm**

**Standard to Profile Regression for Near-Focus Point #3**

$y = 0.008832x + 4.326$

$R^2 = 0.9980$

**Radius = 0.0125mm**
Z-Position (mm)  1/e Waist (mm)  error (mm)  \(y_0\) (mm)  error (mm)
no lens        0.358        0.015       4.288       0.01876
5.44           0.02         0.13        4.328        0.1397
6.44           0.01442      0.0013      4.328        0.00169
7.44           0.01268      0.0053      4.326        0.006539

Table 1: 1/e beam waists and Gaussian center positions calculated by direct fitting of my normalized response data to an algebraic approximation to the cumulative distribution function.

Graphing the Khosrofian and Garetz radii with respect to z position (Figure 6), it is apparent that the beam was progressively leveling off as the z-position was moved upstream.

![1/e Focal Radius vs. Z Position](image)

Figure 6: Summary of the beam radii calculated using Khosrofian & Garetz’s exponent comparison technique. The radii are the blue points. The additional negative reflections and the circles are just there to help visualize the beam that the data implies.

Since the radius measurements do not start to increase again, this suggests that I should have taken one or two more profiles further upstream. However, they are strongly leveling off by this point, so the smallest measured radius of 0.0125mm is probably close to the actual minimum.
This is encouraging because the theoretical radius at focus is 0.0126mm, less than 1% different from either of the smallest measurements.

5. Conclusion

My greatest accomplishments in my capstone research were largely unrelated to the lab's actual experiments. I have contributed the majority of the equipment programming (see Appendix II) and system debugging, which has been instrumental in getting the equipment automated and usable. Except for some very fundamental hardware conflicts, which will have to be resolved by using multiple computers, the lab is ready to take data.

In terms of beam profiling, I was not able to verify that the Nd:YAG signal is Gaussian enough to retain its shape though focus. However, I have been able to execute the knife-edge method on a pen laser, using two different data analysis schemes, which is quite an accomplishment unto itself. The results vary in consistency and suggest that my current error analysis techniques could stand to be refined.
Figure A 1: Excerpt from my lab book describing design and operation parameters for retrofitting a laser pen pointer as a continuous-use table-mounted laser. Essentially this boils down to: make a dummy cell, wire a bench-top DC power supply to it, strap the pen to an optics table mount, and be careful not to supply more than 3V and 25mA.
Appendix II: Control Programs

Although all we have to measure in the capillary waveguide tests is energy in and energy out, we would ideally like to take data at 10Hz, the pulsation frequency of the Nd:YAG pump laser. Doing this by hand is completely impossible, so we have developed LabVIEW-based computer programs to take data and manage equipment settings from a single computer interface. The main program has been developed over both this past summer and the summer of 2007, with significant portions under my authorship.

Before a data taking run, the control program takes parameters from a user interface screen, referred to in LabVIEW as the front panel. When the program is run, it uses these parameters to: 1) prepare the RM-6600 for the desired style of data collection; 2) create and format the file in which data will be stored; 3) capture and analyze the data coming off of the RM6600’s buffer, in real time; and 4) write to the file only usable data of the highest, reasonably attainable quality.

The primary data-taking operation is to read the RM-6600 data buffer. This gives the program a single ASCII string containing the readings of both RJP-465 detectors. Through a series of search-string and numeric comparison operations, the program filters the data for four key problems. First, it checks whether the string actually has data. Provided there is actual information to process, it then splits the original data string into the individual detector readings, A and B. The program then checks both strings individually for the overrange code, a zero, and whether the reading is low enough to warrant lowering the sensitivity range. Boolean operations
are connected to the results of these tests. If any come up as true, the data string is discarded. If necessary the detectors are reranged, and a new reading is taken in its place.
Figure A2: This is the block diagram for the full z-scan program. The green sections are z-scan-specific (the loop of green means that particular loop structure is for z-scanning), while everything else is just the energy.
meter program. Turquoise sections are for generating data files. The yellow section prepares the energy meters. The red section collects energy data. And the purple sections filter energy data.

The exact method and sequence of these tests represents a significant portion the time I spent on this project and is thus worth going over in some detail. From what I can tell, blank buffer reads constituted one of the biggest obstacles to making the program work. The RM 6600’s user manual does not mention what happens when the buffer is read before a new piece of data is sent to it. Not surprising in retrospect, what it apparently produces is some combination of carriage-returns, end-lines, null characters, and/or spaces (we never did figure out exactly which). Because these blank reads contain none of the red-flag substrings of which we were aware (e.g. “OR” or “0.000”), they were simply passed through to the data file, via a general purpose string to floating point converter, which interprets white space as a zero. The bug took so long to identify because it looked like a problem with properly identifying and disposing of legitimate zeroes, when it was in fact something completely different. My solution takes the raw two-detector data string and applies two string searches, one for “OR” and one for “E”. Legitimate buffer content is either out of range or a number in exponential notation, so, if both of the searches return “–1” (meaning no index within the string corresponding to the desired characters), then the read is blank, all other analysis is bypassed, and the program reads the buffer again. The data files generated by the program are now free of zeros, so the program seems to be recording usable data.

The next step is to find out which, if any, of the detectors are reading overrange. To this end, sub-VI’s individually search within the A and B strings for “OR”. If a search for one comes up positive, the command for the next lowest range is sent to the effected detector, the variable for that detector’s range is incremented by one, and a “true” boolean tag is carried forward in that particular iteration of the data read loop.
The A and B strings are now converted to floating point format and immediately tested for equality to zero. There are several reasons for specific concern for zeros. The primary issue is that what we really want is transmission data. Transmission being the quotient of the energy out and energy in readings, we not only have to have coincident numbers for both A and B, but we also cannot use zeros because they generate either a divide by zero error or a jump discontinuity to zero transmission. Important too is that zeros are also almost certainly experimental errors and are not indicative of an underrange reading. Why? First, there is always detectable noise in the room, so, at the most sensitive setting, a functioning detector will never read zero. Second, the detectors are kept in their optimal sensitivity range, which gives 3 to 4 significant figures. The signal would therefore have to drop by at least 3 orders of magnitude within a tenth of a second (between pulses) in order to generate an underrange zero, and this simply does not happen when the laser is functioning properly. What can change the readings by that much is losing read-synchronization with the laser’s trigger (a.k.a. reading between pulses), and, were we to change equipment settings over such a reading, it would certainly not involve the range of the detectors. If a zero is detected, a “true” boolean tag is carried forward in that iteration of the data read loop.

Provided that the reading is non-zero, the data point is sent through a sub-VI that checks for underrange reads. The energy value is checked against the top end of the next range down. Specifically, the user initially specifies a “fluctuation” value in the front panel (default is 5%), and the reading is compared to the maximum for the next range down minus that percent fluctuation (prevents spastically jumping between ranges). If the value is in this sense significantly underrange and that detector is not already bottomed out, the effected detector is sent the command for the next range down, the variable for that detector’s range is decremented
by one, and a “true” boolean tag is carried forward in that particular iteration of the data read loop.

At the end of all of this filtering, two sets of boolean tests are performed. First, all of the tags from overrange and underrange tests are checked for a “true.” If there is a true, the subsequent reranging command has stopped the RM 6600’s data collection, and the “AD” command is re-sent to the RM 6600 to re-initialize data collection. Second, all of the boolean tags are checked for a “true.” If any tag has been tripped, it means that at least one data point is unusable. This means the transmission value would be invalid. Thus the data read loop is reiterated, dropping the current data values and over-writing them with a new buffer read. The data read loop is thus reiterated until a pair of good A and B values are obtained, at which point they are carried to the next loop out and written to the final data file. That outer loop runs the number of times specified by the user in the “ Shots” variable.

Another significant fix involved the program’s automatic sensitivity ranging. Historically, there were two apparently unrelated bugs. First, the program kept ranging up twice in a row on what should have been very low energy test runs. And, second, the light meters were maxing out very quickly. These two bugs were the same problem. Our RM 6600 does not behave exactly according to the manual that we downloaded from the manufacturer’s website. Apparently at some time in the past, Laser Precision Corp. engineers decided to change the hardware’s indexing of the range command. In what appears to be an attempt at back-compatibility for older control software, the RM 6600 that we own can accept both the old and new set of range indices, and it does this by interpreting both RA0 and RA1 as the exact same command. The multiple reranging bug was simply RA1 doing absolutely nothing. Since our manual quotes the old 0-5 set of commands, our program wasn’t even capable of setting the
highest sensitivity, whose command is RA6, which is what caused the meters to max out faster than expected. After fixing this, most of our reranging bugs vanished, and the program now switches range like a charm.

References


